

# ***Science Models: How Uncertainty Drives Design***



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# Bounding $\eta_{\oplus}$ (What is $\eta_u$ ?)

- At a previous SWG, some discussion of how  $\eta_{\oplus}$  impacts mission success
- Statement made that if value of  $\eta_{\oplus}$  is too small, there is no hope of success due to finite mission life
  - But, what if we know exactly which stars to look at? (Precursor science)
- There are two separate issues:
  1. How many stars must we look at if we hope to find 'Earths'
  2. How far must we be able to look to see the 'right' stars
- We want to address the question of how precursor science, as well as uncertainty, impacts system design
- Definitions used:
  - $\eta_{\oplus}$  – Fraction of TOTAL stars that HAVE planets of 'interest' (POI)
  - $\eta_u$  – Fraction of TOTAL stars we THINK have planets of interest. This is the upper bound for  $\eta_{\oplus}$
  - $p$  – Probability of finding POI within a sample of stars
  - $N_s$  – Total number of stars in a volume of space
  - $R$  – Radius of volume of space
  - $N_{s/P}$  – Total number of stars with POI within a volume of space

# Why is $\eta_u$ an important parameter?

- By its definition,  $\eta_u$  must vary between the value of  $\eta_{\oplus}$  and 1

$$\eta_{\oplus} \leq \eta_u \leq 1$$

- If we assume that we are not going to sample stars that are unfavorable, then the probability ( $p$ ) of finding a POI within a sample of stars is

$$p = \frac{\eta_{\oplus}}{\eta_u}$$

- With no a priori knowledge (precursor), this probability is simply  $\eta_{\oplus}$ , but with complete knowledge it could equal 1
- If only 1 in 1000 stars has a POI, but we know which one it is, then we only have to look at one star
  - BUT it still may be **very** far away

- Although  $\eta_u$  factors into how **many** stars we need to search, it does not affect how **far** we must look
- Assuming stars are uniformly distributed:

$$N_{S/\oplus} = \eta_{\oplus} N_S = \eta_{\oplus} \frac{4}{3} \pi R^3 n_S$$

$$R = \left( \frac{3 N_{S/\oplus}}{\pi n_S \eta_{\oplus}} \right)^{1/3}$$

For example, if  $\eta_{\oplus} = 0.001$   
and  $\eta_u = 0.01$  (good knowledge)

We have a 0.1 probability that the stars we look at will have planets, so we don't have to survey as many of them

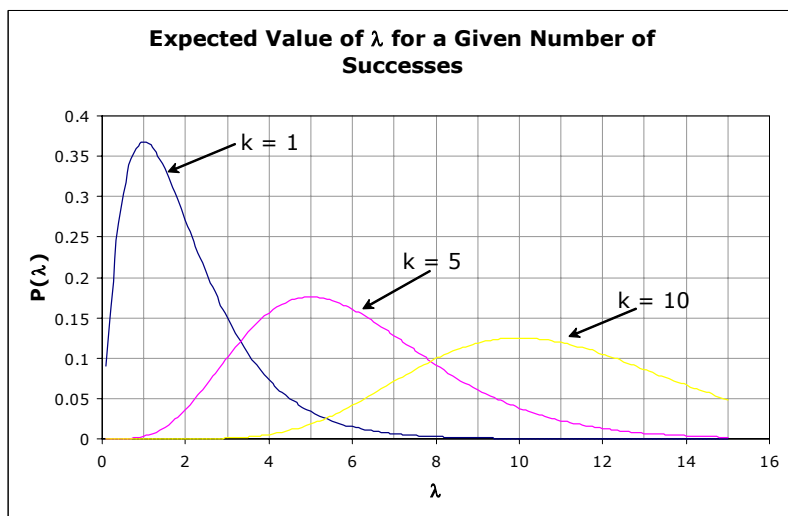
**BUT** we have to look 10x farther to find them

- Impact: May still need larger baselines

# Applying Sampling Statistics

- For large sample size ( $N$ ), the sampling statistics approach a Poisson process

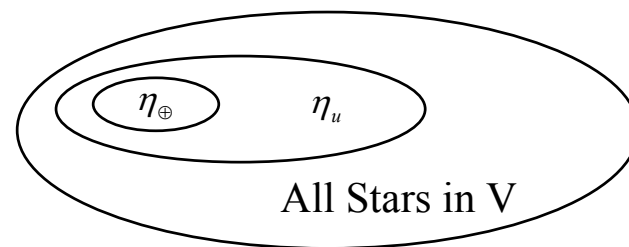
$$P(k) = \frac{\lambda^k}{k!} e^{-\lambda} \quad \lambda = pN$$



- The expected value of  $\lambda$  for the distribution is the number of trial successes
- Given  $N$  samples, and  $k$  successes, the most likely value of  $p$  is  $k/N$
- The uncertainty in  $\lambda$  is smaller with fewer successes

- Only F, G and K type stars are assumed to be likely to have planets of interest
- Currently our 'guess' at  $\eta_u$  is then

$$\eta_u = \frac{N_{F-G-K}}{N_{All}}$$



⇒ We are only considering those stars

- If precursor science 'surveys' 100 F-G-K stars, and 'identifies' 2 POIs, then the most likely value of  $\lambda$  for the sample is 2, and  $p=1/50$
- But, what if we also learn something about why these stars had planets?

# How Can Precursors Impact $\eta_u$ ?

- If precursor observation and theory estimates that 1/50 of F-G-K type stars has POI, then two things might happen:

1) If a notable correlation can be made, then  $\eta_u$  can be **reduced**, and  $p$  **increased**

- E.g. 'All favorable stars are G-type'

2) Otherwise, we don't know how to be more selective in our search, and therefore cannot reduce  $\eta_u$

- Instead, we know that the  $p$  associated with our current search strategy has an expected value of

$$p = \frac{\eta_{\oplus}}{\eta_u} \leq 0.02$$

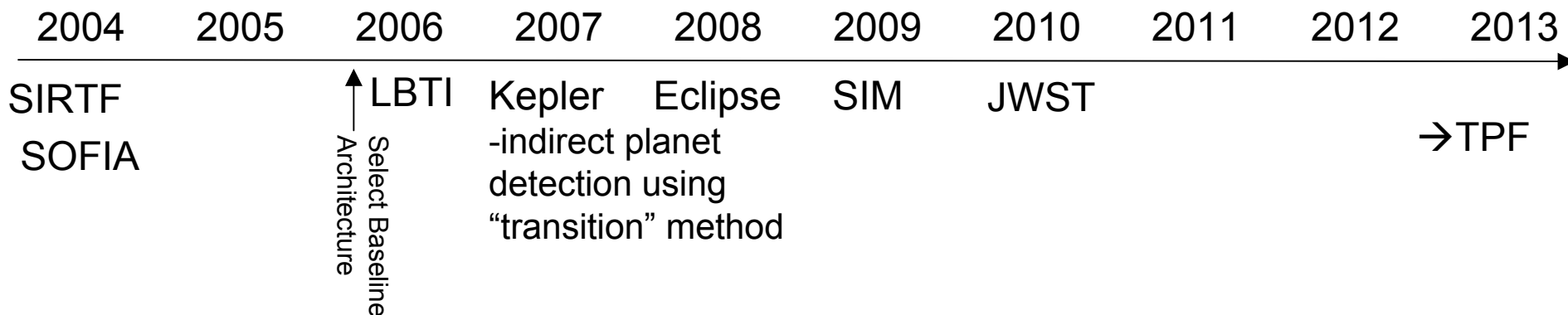
- If  $P_f$  is the likelihood of failing to find any planets in the next sample set, we can calculate

$$N_{samples} = -\frac{1}{p} \ln(P_f)$$

- Given  $p$ , we can determine the number of additional samples that must be taken
- With the current value of  $p = 0.02$ , a likelihood of 1% for failing to find any planets of interest would mean we have to survey >230 stars
- If it is eventually determined that there is a reason why only these 1/50 stars has a POI, then the value of  $\eta_u$  can be reduced by a factor of 1/50, as our future search strategies will be more selective

# When Does $\eta_u$ Change?

- $\eta_u$  changes as a result of processed precursor science mission data
- Data processing will lag launch dates by 2-3 years



- Current Projects
  - Keck (July 2003 – 1<sup>st</sup> published science observation, 2001 – 1<sup>st</sup> light)
    - Exozodiacal dust characterization & nulling
    - Can detect Uranus size planets around stars up to 60 light yrs away
- Future Projects before Architecture Selection
  - SOFIA
    - Proto-planetary disks, planet formation in nearby star systems
  - SIRTF
    - Brown dwarfs, super-planets
    - Dust disks surrounding nearby stars

*Most of the Precursor Science Data Arrives AFTER Baseline Architecture Selection*

# How Does $\eta_u$ Change? (Case Study)

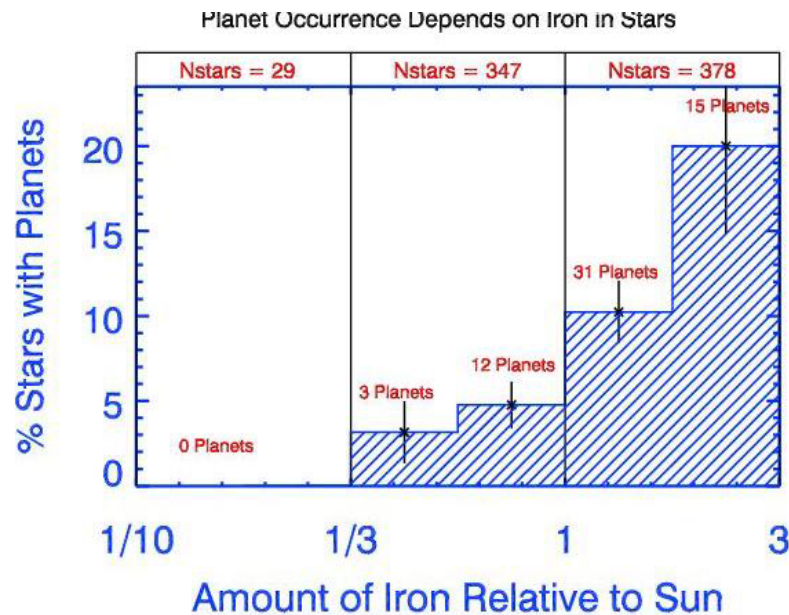
- $\eta_u$  changes with significant precursor science/observation
- **Example:** Correlation between iron-rich stars and large planets\*
- **Scenario** – 754 nearby sun-like stars surveyed to determine the presence of a Jupiter-sized planet & spectrum of each star taken to determine amount of iron
- **Results** - 61 planets existed. Data suggests metal-rich stars are more likely to develop planetary systems
  - Near linear relationship between Fe and planet existing

- How this affects  $\eta_u$  for large planets around sun-like stars:

- Stars with 1/10 to 1/3 the amount of Fe appear to have no detectable planets
- Assuming this data is representative of an arbitrarily large data set

$$\eta_u^{new} = \eta_u^{old} \left( 1 - \frac{29}{754} \right)$$

- Incorporating the weighted probabilities is work in progress



\*Fischer, UCBerkeley/Valenti, STScI, <http://exoplanets.org/metallicity.html>

# Limits of a Fixed Baseline System

- The time required to do detection about a given star is the same regardless of habitable zone coverage (one aspect of ‘completeness’)
  - Less than 100% coverage allows possibility of missing a planet that is there
- Fixed baseline systems will have 100% coverage over some stars, but partial coverage over most.
  - Stars with 100% coverage occur in a narrow distance band
  - A larger number of stars can be included in this band by looking farther out, but at the expense of longer integration times (fewer stars actually viewed)



	BL 1	BL 2	BL 3
# of stars in $\Delta R$ ( $C=1$ )	100	500	1000
# viewable over mission	50	40	30
# of planets if $\eta_{\oplus} = 0.01$	1	5	10
$P_f$ if $\eta_{\oplus} = 0.01$ , $\eta_u = 1$	60%	67%	74%
# of planets if $\eta_{\oplus} = 0.001$	0	0	1
$P_f$ if $\eta_{\oplus} = 0.001$ , $\eta_u = .01$	100%	100%	5%

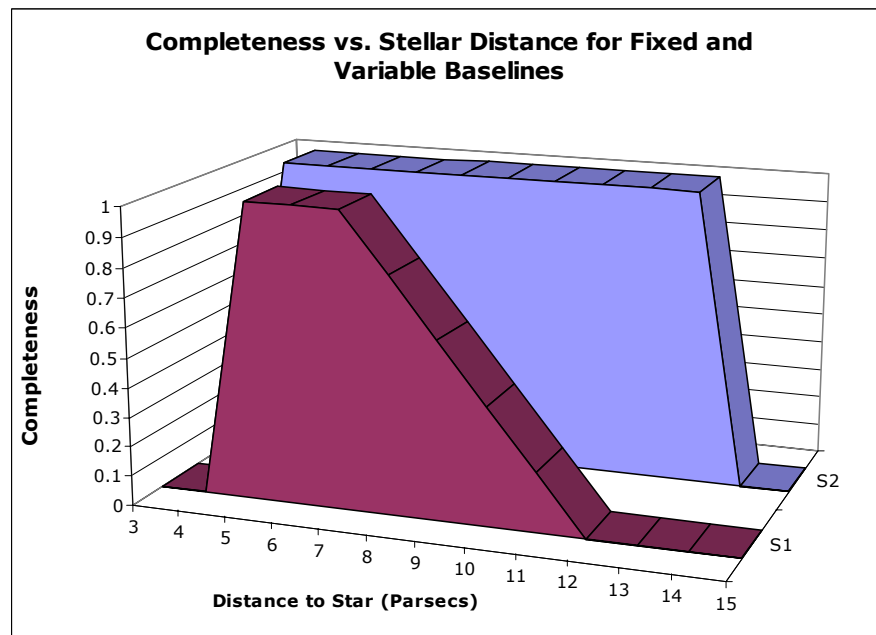
- Higher uncertainty in which stars should be observed favors looking closer, since a larger sample set can be viewed over the mission lifetime
  - This is the effect of distance on integration time
- However, if  $\eta_{\oplus}$  is much smaller than predicted, the number of stars accessible by a shorter fixed baseline system may be insufficient to have an ‘Earth’
- Longer fixed baseline systems are less likely to find Earths if  $\eta_u$  is large, but may be the only chance of finding them if precursor missions reduce  $\eta_u$

*Because precursor science data will lag the architecture downselect there is no way to know whether a shorter or longer fixed baseline is better*



# Merits of a Variable Baseline

- Unlike a fixed baseline system, a variable baseline allows for observational completeness regardless of the stellar distance
- If  $\eta_u$  turns out to be relatively large, operating at shorter baselines will allow for many more observations to be made over the mission lifetime, increasing science throughput
- If  $\eta_u$  turns out to be relatively small, operating at longer baselines grants access to a much larger volume of stars, increasing science throughput
- If  $\eta_u$  turns out to be very small, then very large baselines may be necessary to access a sufficient volume of space to insure the stars with planets of interest can be observed
  - The longer integration times make the need for a tunable baseline (to achieve full completeness) even more important



*Independent of technological issues, for or against, a formation flown variable baseline system appears to offer the lowest risk for science return*

# Summary

- Even if  $\eta_u$  is very small, precursor science will allow us to upper bound its value in such a way that we can be smarter about which stars are observed
- A large uncertainty in  $\eta_u$  drives us toward a desire for more observations, and therefore shorter baselines
  - The danger lies in the possibility that the limited number of stars accessible by a shorter fixed baseline system may no have any 'Earths'
- Larger fixed baseline systems allow for more stars to be accessed, but without good precursor knowledge, we cannot effectively limit our search
- The problem is that the architecture downselect occurs before it will be known whether searching closer or farther is the better approach
- The most robust solution is to make the baseline variable, over a relatively large extent, allowing operation in either mode
  - This supports a formation flown system as providing the least science risk